Unit-IV (Optimization)

Linear Optimization

Linear Optimization (also called Linear Programming – LP) is a mathematical technique used to determine the best possible outcome (such as maximum profit or minimum cost) in a given situation with limited resources. It is widely used in business decision-making, including production planning, transportation, finance and resource allocation.

A linear optimization problem consists of:

- **Decision Variables**
- **Objective Function (Goal)**
- **Constraints (Restrictions)**

Goals (Objective Function)

The **goal** of a linear optimization problem is expressed through an **objective function**, which is a linear equation representing what needs to be maximized or minimized.

Types of Goals:

1. **Maximization**
    - Example: Maximize profit, maximize revenue and Maximize output.
2. **Minimization**
    - Example: Minimize cost, minimize time and Minimize resource usage.

Example Objective Function:

Let (x) = number of units of Product A and (y) = number of units of Product B

Profit is Product A = ₹50 per unit and Product B = ₹40 per unit

Objective Function:
Maximize Z = 50x + 40y Here, (Z) represents total profit.

Constraints in Linear Optimization

Constraints are the limitations or restrictions on decision variables. These can include:

- Resource availability (labour, materials, time)
- Budget limits
- Production capacity

Constraints are expressed as linear inequalities or equations.

Example Constraints:

1. **Labor Constraint:** A labour constraint limits production or activity based on the availability of workforce (labour hours or workers).
2. **Material Constraint:** A material constraint limits production based on the availability of raw materials or resources.
3. **Non-negativity Constraint:** These ensure that production quantities cannot be negative.

Steps to Formulate a Linear Optimization Problem

1. **Identify Decision Variables:**
    - What needs to be decided? (e.g., number of products)
2. **Formulate Objective Function**
    - What is the goal? (maximize/minimize)
3. **Identify Constraints**
    - What limitations exist?
4. **Solve the Model**
    - Graphical method (for 2 variables)
    - Simplex method, (for large problems)

Example on Business Problem: Production Optimization

Problem Statement:

A company produces two products: A and B. The company wants to maximize profit under resource constraints.

Resource	Product A	Product B	Available
Labor	2 hrs	1 hr	100 hrs
Material	1 unit	2 units	80 units

Applications of Optimization:

- ❖ Manufacturing Production planning
- ❖ Supply chain management
- ❖ Transportation problems
- ❖ Workforce scheduling